

The gluonic condensate from the hyperfine splitting

$M_{\text{cog}}(\chi_{cJ}) - M(h_c)$ in charmonium

A.M. Badalian

State Research Center, Institute of Theoretical and Experimental Physics, Moscow, Russia

B.L.G. Bakker

Vrije Universiteit, Department of Physics, Amsterdam, The Netherlands

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Abstract

The precision measurement of the hyperfine splitting $\Delta_{\text{HF}}(1P, c\bar{c}) = M_{\text{cog}}(\chi_{cJ}) - M(h_c) = -0.5 \pm 0.4$ MeV in the Fermilab–E835 experiment allows to determine the gluonic condensate G_2 with high accuracy if the gluonic correlation length T_g is fixed. In our calculations the negative value of $\Delta_{\text{HF}} = -0.3 \pm 0.4$ MeV is obtained only if the relatively small $T_g = 0.16$ fm and $G_2 = 0.065(3)$ GeV⁴ are taken. These values correspond to the “physical” string tension ($\sigma \approx 0.18$ GeV²). For $T_g \geq 0.2$ fm the hyperfine splitting is positive and grows for increasing T_g . In particular for $T_g = 0.2$ fm and $G_2 = 0.041(2)$ GeV⁴ the splitting $\Delta_{\text{HF}} = 1.4(2)$ MeV is obtained, which is in accord with the recent CLEO result.

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I. INTRODUCTION

Recently, the CLEO Collaboration has presented preliminary results on the mass of the $h_c(1^1P_1)$ resonance in the reaction $\psi' \rightarrow \pi^0 h_c$ [1]. It appeared that the central value of the observed mass, $M(h_c) = 3524.4 \pm 0.9$ MeV, is about 1.5 MeV lower than the one found in the recent Fermilab–E835 experiment, in the $p\bar{p} \rightarrow h_c \rightarrow \eta_c \gamma \rightarrow 3\gamma$ reactions, where $M(h_c) = 3525.8 \pm 0.4$ MeV is measured [2]. Consequently, the central values of the hyperfine (HF) splittings, $\Delta_{\text{HF}}(1P, c\bar{c}) = M_{\text{cog}}(\chi_{cJ}) - M(h_c)$, corresponding to the measured masses $M(h_c)$, have different signs:

$$\Delta_{\text{HF}}^{(1)}(\text{exp}) = -0.5 \pm 0.4 \text{ MeV (E835)}, \quad (1.1)$$

$$\Delta_{\text{HF}}^{(2)}(\text{exp}) = +0.96 \pm 1.03 \text{ MeV (CLEO)}, \quad (1.2)$$

although they are compatible within 2σ . (In Eqs. (1.1, 1.2) the value $M_{\text{cog}}(\chi_{cJ}) = 3525.32 \pm 0.13$ MeV is taken from PDG [3]). The small numerical difference between these experimental values makes an essential difference for the theoretical interpretation, as we will show in this paper.

Already many years ago it was understood that the sign and small magnitude of $\Delta_{\text{HF}}(1P)$ in charmonium occur due to the almost equal (and small) contributions from the perturbative (P) and nonperturbative (NP) HF interactions [4]:

$$\Delta_{\text{HF}}(1P) = \Delta_{\text{HP}}^P(1P) + \Delta_{\text{HF}}^{\text{NP}}(1P), \quad (1.3)$$

where the perturbative contribution is always negative [5, 6]. Just due to the cancellation between the negative perturbative contribution and the positive NP one, the value $\Delta_{\text{HF}}(1P, c\bar{c}) \cong -1$ MeV has been calculated in [6] for a value of the gluonic condensate $G_2 \cong 0.042 \text{ GeV}^4$, while in Ref. [7] the same HF splitting has been obtained for the significantly smaller value $G_2 \approx 0.02 \text{ GeV}^4$. The reason behind this difference will be explained below and comes from the fact that the gluonic condensate actually enters $\Delta_{\text{HF}}^{\text{NP}}(1P)$ in the combination $G_2 T_g^2$ (T_g is the gluonic correlation length) and therefore the extracted value of G_2 depends also on the correlation length T_g used. Unfortunately, at present there is a large uncertainty in the value of the gluonic condensate, even in the framework of the same approach, like QCD sum rules. Values ranging from $G_2 = 0.012 \text{ GeV}^4$ up to $G_2 \cong 0.07(3) \text{ GeV}^4$ are used [8, 9].

The new experimental value of the h_c mass [2] (measured with an accuracy of 0.4 MeV) allows one to determine G_2 with better precision. In this paper we show why a precise knowledge of $\Delta_{\text{HF}}(1P)$ in charmonium is so important for a fundamental theory and explicitly extract the gluonic condensate G_2 from existing experimental data.

II. THE PERTURBATIVE HF INTERACTION

We consider here the perturbative HF interaction in one-loop approximation which is well known. The splitting $\Delta_{\text{HF}}(1P)$ is given by the expression [5]:

$$\Delta_{\text{HF}}^P(1P) = \frac{8}{9} \left[\frac{1}{4} - \frac{n_f}{3} \right] \frac{\alpha_s^2(\mu)}{\pi} \frac{1}{m_q^2} \langle r^{-3} \rangle_{1P}. \quad (2.1)$$

It is important to take the HF splitting just in the form (2.1) while another (approximate) definition suggested in Ref. [5],

$$\Delta_{\text{HF}}^P(1P) = \frac{10}{81} \left[\frac{1}{4} - \frac{n_f}{3} \right] \frac{\alpha_s}{\pi} \left\{ M(^3P_2) - M(^3P_0) \right\}, \quad (2.2)$$

cannot be used because it is valid only in lowest order and neglects second order corrections ($\sim \alpha_s^2/\pi$) and the NP contribution. It will be shown later that the contribution of the neglected terms to the difference $M(\chi_{c2}) - M(\chi_{c0}) = 3a + 0.9c$ (where a and c are the spin-orbit and the tensor splitting) is about 26% and has negative sign (see the numbers in Table I).

It is also important that if relativistic corrections are taken into account, the current (pole) quark mass in (2.1) must be replaced by the average kinetic energy, $\omega_q = \langle \sqrt{\mathbf{p}^2 + m_q^2} \rangle_{1P}$, (usually called the constituent quark mass). This modification can be rigorously derived within the Fock-Feynman-Schwinger representation of the gauge-invariant meson Green function, when the spin-dependent interaction can be considered as a perturbation [10]. Then instead of (2.1) one must use

$$\Delta_{\text{HF}}^P(1P) = -\frac{26}{27} \frac{\alpha_s^2(\mu)}{\pi} \frac{1}{\omega_q^2} \langle r^{-3} \rangle_{1P} \quad (n_f = 4). \quad (2.3)$$

As seen from Eq. (2.3) the HF splitting strongly depends on the coupling $\alpha_s(\mu)$, where the scale μ in $\alpha_s(\mu)$ cannot be arbitrary. It is clear that for the 1P_1 state the scale should be the same as in the coupling $\alpha_{\text{FS}}(\mu)$ used in the fine structure (FS) splittings of the $\chi_c(^3P_J)$

mesons. Fortunately, $\alpha_{\text{FS}}(\mu)$ can be directly extracted from the experimental values of the spin-orbit and tensor splittings: as derived in [11] the following relation is valid

$$\alpha_{\text{FS}}^2(\mu) = \frac{\pi\omega_q^2\{\eta_c(\text{exp}) - |a_{\text{NP}}(1P)|\}}{2f_4(1P)}. \quad (2.4)$$

Here $\eta_c(\text{exp}) = \frac{3}{2}c(\text{exp}) - a(\text{exp}) = 0.024(1)$ GeV; the tensor splitting $c(\text{exp}) = 0.039(1)$ GeV and the spin-orbit splitting $a(\text{exp}) = 0.0346(2)$ GeV are calculated from the χ_{cJ} masses.

In Eq. (2.4) $a_{\text{NP}}(1P)$ is the Thomas precession (NP) term:

$$a_{\text{NP}}(1P) = -\frac{\sigma}{2\omega_q^2}\langle r^{-1} \rangle_{1P}, \quad (2.5)$$

while f_4 is the matrix element (m.e.) entering the second order perturbative part of the parameter η_P :

$$\eta_P = \frac{3}{2}c_P - a_P = \frac{3}{2}c_P^{(2)} - a_P^{(2)} = \frac{2\alpha_{\text{FS}}^2(\mu)}{\pi\omega_q^2}f_4. \quad (2.6)$$

For the extraction of the strong coupling constant it is essential that the m.e. f_4 does not explicitly depend on the renormalization scale μ :

$$f_4 = 1.97834\langle r^{-3} \rangle_{1P} - \langle r^{-3} \ln(\bar{m}r) \rangle_{1P} \quad (n_f = 4). \quad (2.7)$$

The derivation of the HF interaction in Ref. [5] shows that the mass $\bar{m}(\bar{m})$, entering f_4 , is the current (Lagrangian) mass ($\bar{m} = 1.16$ GeV and m_c (pole)=1.45 GeV are taken here).

The analysis in Ref. [11] has shown that the extracted value of $\alpha_{\text{FS}}(\mu_{\text{FS}})$ (in the \overline{MS} scheme) and the scale μ_{FS} significantly differ if the constituent mass ω_c (about $1.6 \div 1.7$ GeV) instead of the pole mass m_c (about $1.4 \div 1.48$ GeV) enters the expression (2.4). For example, the precise description of the FS splitting (with an accuracy better than 1%) for $m_c(\text{pole}) = 1.45$ GeV in Eq. (2.4) gives

$$\alpha_{\text{FS}}(\mu_1) = 0.358 \text{ with } \mu_1 = 0.70 \text{ GeV}. \quad (2.8)$$

In relativistic calculations with $\omega_c(1P) = \langle \sqrt{\mathbf{p}^2 + m_c^2} \rangle_{1P} \cong 1.66$ GeV the value of the coupling constant $\alpha_{\text{FS}}(\mu_R)$ extracted from Eq. (2.4) appears to be significantly larger,

$$\alpha_{\text{FS}}(\mu_R) = 0.514; \quad \mu_R = 0.51 \text{ GeV}. \quad (2.9)$$

From our point of view it is the second choice which should be preferred, because the value $\alpha_{\text{FS}}(\mu_1)$ in Eq. (2.8) is too small for such a small scale as $\mu_1 = 0.70$ GeV. It is known that

TABLE I: The spin-orbit and tensor splittings (in MeV) of the χ_{cJ} mesons (the static potential is taken from [13] with the parameters $m_c(\text{pole}) = 1.45$ GeV, $\omega_c = 1.66$ GeV, $\sigma = 0.18$ GeV², $\Lambda_{\overline{MS}}^{(4)}(2\text{-loop}) = 267$ MeV, $\alpha_{\text{FS}}(\mu_{\text{FS}} = 0.51 \text{ GeV}) = 0.514$.

	1st order term	2d order term	NP term
$a(\text{tot}) = 34.58$ $a(\text{exp}) = 34.56(19)$	$a_p^{(1)} = 51.92$	$a_p^{(2)} = -4.24$	$a_{\text{NP}} = -13.10$
$c(\text{tot}) = 39.12$ $c(\text{exp}) = 39.12(62)$	$c_p^{(1)} = 34.61$	$c_p^{(2)} = 4.51$	$c_{\text{NP}} < 1 \text{ MeV}^a)$

^{a)} The reasons why $c_{\text{NP}}(1P)$ is small, are discussed in [11, 12].

the strong coupling in the \overline{MS} scheme is already rather large at the scale $M_\tau = 1.777$ GeV ($\alpha_s(M_\tau) \gtrsim 0.33$ [3]), while at the scale $\mu_{\text{FS}} \cong 1.0$ GeV the coupling $\alpha_{\text{FS}}(1.0 \text{ GeV}) \cong 0.40$ has been obtained in [12] in the analysis of FS splittings of the $2P$ state in bottomonium. Therefore the values given in Eq. (2.9) will be taken here. They result in the following FS splittings of the χ_{cJ} mesons (with an accuracy better than 1%):

$$\begin{aligned} c(1P) &= 39.12 \text{ MeV}, \quad c_{\text{exp}}(1P) = 39.12 \pm 0.62 \text{ MeV}, \\ a(1P) &= 34.58 \text{ MeV}, \quad a_{\text{exp}}(1P) = 34.56 \pm 0.19 \text{ MeV}. \end{aligned} \quad (2.10)$$

The first- and second-order perturbative and NP terms in the FS splittings: $a(1P) = a_p^{(1)} + a_p^{(2)} + a_{\text{NP}}$, $c(1P) = c_p^{(1)} + c_p^{(2)}$ (c_{NP} is very small [11]), are given in Table I.

Having obtained a precise description of the FS of the χ_{cJ} mesons, we can expect that the HF splitting (for the same set of physical parameters) is also determined with good accuracy. From the expression (2.3) (with $\omega_c = 1.66$ GeV, $\alpha_{\text{HF}}(\mu_R) = \alpha_{\text{FS}}(\mu_R = 0.51 \text{ GeV}) = 0.514$, $\langle r^{-3} \rangle = 0.139$) it follows that in charmonium the perturbative contribution has the value

$$\Delta_{\text{HF}}^{\text{P}}(1P) = -4.1 \text{ MeV}, \quad (2.11)$$

which is five times larger than the one obtained in the experiments [1, 2] (in the CLEO experiment [1] Δ_{HF} is positive, 1.0 ± 1.0 MeV).

The matrix elements Eq. (2.7) are calculated here utilizing the solutions of the spinless Salpeter equation with a static potential–linear plus gluon-exchange term, where in two-loop vector coupling the asymptotic freedom behavior at small distances and the freezing of the

coupling at large distances are taken into account. The most important matrix elements are $\langle r^{-1} \rangle = 0.405 \text{ GeV}$, $\langle r^{-3} \rangle_{1P} = 0.139 \text{ GeV}^3$, and $\langle r^{-3} \ln(\bar{m}r) \rangle = 0.095 \text{ GeV}^3$. Note that the m.e. $\langle r^{-3} \rangle$ in the relativistic case is about 30% larger than in nonrelativistic calculations.

III. THE NONPERTURBATIVE HF INTERACTION

Spin-dependent NP potentials have been introduced in [14, 15]. With the use of the vacuum correlation function (v.c.f.) $D(x)$ the HF interaction is written as

$$V_{\text{HF}}^{\text{NP}} = \frac{1}{3\omega_c^2} V_4^{\text{NP}}(r) \quad (3.1)$$

with

$$V_4^{\text{NP}}(r) = 6 \int_0^\infty d\nu D(\sqrt{r^2 + \nu^2}). \quad (3.2)$$

The contribution of the other correlator, $D_1(x)$, has been neglected in Eq. (3.2), because in the unquenched case $D_1(x)$ is small, even compatible with zero [16]. The v.c.f. $D(x)$ is shown to behave as an exponential: $D(x) = d \exp(-x/T_g)$ at $x \gtrsim 0.2 \text{ fm}$ [16, 17] (T_g is the gluonic correlation length), while at smaller $x < r_0$ ($r_0 \lesssim 0.2 \text{ fm}$) $D(x)$ should have a plateau to satisfy the necessary conditions $\frac{d^2 D(x)}{dx^2} \Big|_{x=0} < 0$ and $\frac{dD}{dx} \Big|_{x=0} = 0$, established in [18].

The value of $D(x)$ at the origin, $D(0)$ is related to the gluonic condensate $G_2 = \frac{\alpha_s}{\pi} \langle F_{\mu\nu}(0) F_{\mu\nu}(0) \rangle$:

$$D(0) = \frac{\pi^2}{18} G_2, \quad (3.3)$$

and we assume that $D(r_0) \cong D(0)$, so the factor d in front of the exponent is $d = D(0) \exp(r_0/T_g)$. At the origin the HF interaction Eq. (3.1) has the value $V_{\text{HF}}^{\text{NP}}(r=0) = \pi^2 G_2 (r_0 + T_g) / (9\omega_c^2)$.

The string tension in the confining potential is defined through the same v.c.f. $D(x)$:

$$\sigma = 2 \int_0^\infty d\lambda \int_0^\infty d\nu D(\sqrt{\lambda^2 + \nu^2}) \quad (3.4)$$

and for the form adopted for $D(x)$ we find

$$\sigma = \frac{\pi^3}{18} G_2 T_g^2 \left[1 + \frac{r_0}{T_g} + \frac{1}{2} \left(\frac{r_0}{T_g} \right)^2 \right]. \quad (3.5)$$

From this relation and taking the “physical” value $\sigma \approx 0.178(8) \text{ GeV}^2$ one can determine the gluonic condensate for different values of T_g . However, the gluonic correlation length is

not known with good accuracy and at present different values (between 0.16 fm and 0.26 fm) have been obtained in lattice QCD [16, 17] and in the vacuum correlator method [19]. For comparison we use here two values, $T_g = 0.16$ fm from Ref. [19] and 0.2 fm. We do not consider here the value $T_g \approx 0.3$ fm, because our analysis shows that for such a value of T_g the NP contribution to the HF splitting is too large ($\Delta_{\text{HF}}^{\text{NP}}(1P) \approx 10$ MeV) so that the total HF splitting is positive, about 6 MeV, i.e. several times larger than the experimental value given in Eq. (1.2).

Then from the relation (3.5) with $\sigma = 0.178(8)$ GeV² it follows that

$$\begin{aligned} G_2 &= 0.065(3) \text{ GeV}^4 \quad (r_0 = T_g = 0.16 \text{ fm}), \\ G_2 &= 0.041(2) \text{ GeV}^4 \quad (r_0 = T_g = 0.20 \text{ fm}). \end{aligned} \quad (3.6)$$

The HF splitting, calculated for the interaction (3.2), reduces to the expression [7],

$$\Delta_{\text{HF}}^{\text{NP}}(1P) = \frac{\pi^2 G_2}{9 \omega_c^2} (r_0 + T_g) J, \quad J = \left\langle r K_1 \left(\frac{r}{T_g} \right) \right\rangle_{1P}. \quad (3.7)$$

The accuracy of this approximation is better than 5%. The matrix element J , $J(T_g = 0.16 \text{ fm}) = 0.092 \text{ GeV}^{-1}$ and $J(T_g = 0.20 \text{ fm}) = 0.17 \text{ GeV}^{-1}$, strongly depends on T_g . Then taking G_2 from (3.6) one obtains

$$\begin{aligned} \Delta_{\text{HF}}^{\text{NP}}(1P) &= 3.8(4) \text{ MeV} \quad (T_g = 0.16 \text{ fm}), \\ \Delta_{\text{HF}}^{\text{NP}}(1P) &= 5.5(3) \text{ MeV} \quad (T_g = 0.20 \text{ fm}). \end{aligned} \quad (3.8)$$

Thus the magnitude of the NP contribution (3.8) appears to be larger (smaller) than the perturbative term (2.11) for larger (smaller) gluonic correlation length. Therefore the total HF splitting Eq. (1.3) has different signs for $T_g = 0.16$ fm and $T_g = 0.2$ fm,

$$\Delta_{\text{HF}}(1P) = -0.3(4) \text{ MeV} \quad (T_g = 0.16 \text{ fm}) \quad (3.9)$$

$$\Delta_{\text{HF}}(1P) = +1.4(2) \text{ MeV} \quad (T_g = 0.20 \text{ fm}). \quad (3.10)$$

It is amusing to notice that the HF splitting for $T_g = 0.16$ fm exactly coincides with the experimental value obtained in the E835 experiment [2], while the positive splitting (3.10) is close to the value obtained in the CLEO experiment [1]. To distinguish between these two possibilities $\Delta_{\text{HF}}(1P)$ needs to be measured in charmonium with a better accuracy.

IV. CONCLUSIONS

Our analysis has shown that the “physical” value of the string tension cannot unambiguously fix the gluonic condensate and only the product $G_2 T_g^2$ can be determined.

The HF splitting between the c.o.g of the χ_{cJ} mesons and $h_c(^1P_1)$ provides an additional opportunity to extract the gluonic condensate with good accuracy, due to the cancellation between the negative perturbative and positive NP contributions which both have small magnitude.

To calculate the HF splitting for the $1P$ states in charmonium we use exactly the same coupling $\alpha_s(\mu)$ and matrix elements as in the fine structure analysis of the χ_{cJ} mesons where a high accuracy ($\lesssim 1\%$) is reached. Therefore we estimate the accuracy of our calculations of $\Delta_{\text{HF}}(1P)$ to be equal to 0.3 MeV (0.4 MeV) for the gluonic correlation length $T_g = 0.16$ fm ($T_g = 0.2$ fm).

An additional restriction is also put on the gluonic condensate—it should correspond to the “physical” string tension, $\sigma \cong 0.18 \text{ GeV}^2$, used in the static potential.

Then a negative central value of $\Delta_{\text{HF}}(1P) = -0.3 \text{ MeV}$, as in the E835 experiment [2], is obtained for $G_2 = 0.065 \text{ GeV}^4 (T_g = 0.16 \text{ fm})$. For the larger correlation length, $T_g = 0.2 \text{ fm}$, and the smaller value $G_2 = 0.041 \text{ GeV}^4$ the HF splitting appears to be positive, $\Delta_{\text{HF}}(1P) = +1.4 \text{ MeV}$. So it is of prime importance for the deduction of the values of the gluonic condensate and correlation length that the discrepancy between the two experimental results be removed.

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